Query Execution Techniques in PostgreSQL

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Goals

- Describe how Postgres works internally
- Shed some light on the black art of EXPLAIN reading
- Provide context to help you when tuning queries for performance

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- Shed some light on the black art of EXPLAIN reading
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Outline

- The big picture: the roles of the planner and executor
- Plan trees and the Iterator model
- Scan evaluation: table, index, and bitmap scans
- Join evaluation: nested loops, sort-merge join, and hash join
- S Aggregate evaluation: grouping via sorting, grouping via hashing
- 6 Reading EXPLAIN

Typical Query Lifecycle

Parser: analyze syntax of query $query \ string \Rightarrow Query \ (AST)$ Rewriter: apply rewrite rules (incl. view definitions) Query $\Rightarrow \ zero \ or \ more \ Query$ Planner: determine the best way to evaluate the query Query $\Rightarrow \ Plan$ Executor: evaluate the query $Plan \Rightarrow PlanState$ $PlanState \Rightarrow \ query \ results$

Query Planner

Why Do We Need A Query Planner?

- Queries are expressed in a logical algebra (e.g. SQL)
 - "Return the records that satisfy"
- Queries are executed from a physical algebra (query plan)
 - "Index scan table x with key y, sort on key z,"
- For a given SQL query, there are many equivalent query plans
 - Join order, join methods, scan methods, grouping methods, order of predicate evaluation, semantic rewrites, ...
- Difference in runtime cost among equivalent plans can be enormous

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Two Basic Tasks of the Planner

- Enumerate the set of plans for a given query
- Stimate the cost of executing a given query plan

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Query Plans

- The operators of the physical algebra are the techniques available for query evaluation
 - Scan methods, join methods, sorts, aggregation operations, ...
 - No simple relationship between logical operators and physical operators
- Each operator has 0, 1 or 2 input relations, and 1 output relation
 - 0 inputs: scans
 - 2 inputs: joins, set operations
 - 1 input: everything else
- The operators are arranged in a tree
 - Data flows from the leaves toward the root
 - The "query plan" is simply this tree of operators
 - The output of the root node is the result of the query

Example Query Plan



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Conceptual Plan Tree Structure

From leaf \rightarrow root, a typical query plan *conceptually* does:

- Scans: heap & index scans, function scans, subquery scans, ...
- 2 Joins
- Grouping, aggregation and HAVING
- Sorting (ORDER BY)
- Set operations
- Projection (apply target list expressions)

In practice, various reordering and rewriting games, such as:

- Pushdown: move operators closer to leaves to reduce data volume
- Pullup: transform subqueries into joins
- Choose lower-level operators to benefit upper-level operators

Common Operator Interface

Most Postgres operators obey the same interface for exchanging data:

Init(): acquire locks, initialize state

GetNext(): return the next output tuple

- Typically calls GetNext() on child operators as needed
- Blocking operation
- Optionally supports a *direction* (forward or backward)

ReScan(): reset the operator to reproduce its output from scratch

MarkPos(): record current operator position (state)

RestorePos(): restore previously-marked position

End(): release locks and other resources

A Clean Design

- Encodes both data flow and control flow
- Operators simply pull on their inputs and produce results
- Encapsulation: each operator needs no global knowledge

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Disadvantages

- 1 tuple per GetNext() is inefficient for DSS-style queries
- Operators can only make decisions with local knowledge
- Synchronous: perhaps not ideal for distributed or parallel DBMS

Pipelining

What Is Pipelining?

How much work must an operator do before beginning to produce results?

- Some operators must essentially compute their entire result set before emitting any tuples (e.g. external sort): "materialization"
- Whereas other, pipelinable operators produce tuples one-at-a-time

Pipelining

What Is Pipelining?

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Why Is It Important?

- Lower latency
- The operator may not need to be completely evaluated
 - e.g. cursors, IN and EXISTS subqueries, LIMIT, etc.
- Pipelined operators require less state
 - Since materialized state often exceeds main memory, we may need to buffer it to disk for non-pipelined operators
- Plans with low startup cost sometimes > those with low total cost

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- Disk I/O dominates query evaluation cost
- $\bullet\,$ Random I/O is more expensive than sequential I/O
 - $\bullet \ \ldots$ unless the I/O is cached
- Reduce inter-operator data volume as far as possible
 - Apply predicates as early as possible
 - Assumes that predicates are relatively cheap
 - Also do projection early
 - TODO: pushdown grouping when possible
- Fundamental distinction between plan-time and run-time
 - Planner does global optimizations, executor does local optimizations
 - $\bullet~$ No feedback from executor $\rightarrow~$ optimizer

Sequential Scans

- Simply read the heap file in-order: sequential I/O
 - Doesn't necessarily match on-disk order, but it's the best we can do
- Must check heap at some point anyway, to verify that tuple is visible to our transaction ("tqual")
- Evaluate any predicates that only refer to this table

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The Problem

Must scan entire table, even if only a few rows satisfy the query

Basic Idea

Use a secondary data structure to quickly find the tuples that satisfy a certain predicate

• Popular index types include trees, hash tables, and bitmaps

Downsides

- More I/Os needed: 1 or more to search the index, plus 1 to load the corresponding heap page
 - Postgres *cannot* use "index-only scans" at present
- \bullet Random I/O needed for both index lookup and heap page
 - Unless the index is *clustered*: index order matches heap order
- Therefore, if many rows match predicate, index scans are inefficient
- Index must be updated for every insertion; consumes buffer space

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Illustration



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The Canonical Disk-Based Index For Scalar Data

- \bullet On-disk tree index, designed to reduce # of disk seeks
 - 1 seek per tree level; therefore, use a high branching factor: typically 100s of children per interior node
 - B != "binary"!
 - All values are stored in leaf nodes: interior nodes only for navigation
 - Tree height $O(\log_{100} n)$: typically 5 or 6 even for large tables
 - Therefore, interior nodes are often cached in memory
- Allows both equality and range queries: \leq , <, >, \geq , =
 - Leaf nodes are linked to one another
- Highly optimized concurrent locking scheme
- "Ubiquitous" even in 1979

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Bitmap Scans

Basic idea: decouple scanning indexes from scanning the heap

- Is For each relevant index on the target table:
 - Scan index to find qualifying tuples
 - Record qualifying tuples by setting bits in an in-memory bitmap
 - 1 bit per heap tuple if there is space; otherwise, 1 bit per heap page
- ② Combine bitmaps with bitwise AND or OR, as appropriate
- Use the bitmap to scan the heap in order

Benefits

- Reads heap sequentially, rather than in index order
- Allows the combination of multiple indexes on a single table
 - More flexible than multi-column indexes

Importance

Join performance is key to overall query processing performance

Much work has been done in this area

Toy Algorithm

To join *R* and *S*:

• Materialize the Cartesian product of R and S

• All pairs (r, s) such that $r \in R, s \in S$

2 Take the subset that matches the join key

... laughably inefficient: $O(n^2)$ space

Basic Algorithm

For a NL join between R and S on R.k = S.k:

```
for each tuple r in R:
   for each tuple s in S with s.k = r.k:
      emit output tuple (r,s)
```

Terminology: R is the outer join operand, S is the inner join operand. Equivalently: R is left, S is right.

Simplest Feasible Algorithm

Only useful when finding the qualifying R and S tuples is cheap, and there are few such tuples

- R and S are small, and/or
- Index on *R.k*, join key (or other predicates) is selective

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Basic Algorithm

```
For a SM join between R and S on R.k = S.k:
sort R on R.k
sort S on S.k
forboth r in R, s in S:
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(Duplicate values make the actual implementation more complex.)

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The Problem

- This works fine when both *R* and *S* fit in memory
- ... unfortunately, this is typically not the case

Avoid An Explicit Sort

- Traversing leaf level of a B+-tree yields the index keys in order
- Produce sorted output by fetching heap tuples in index order
- NB: We can't use a bitmap index scan for this purpose!

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Downsides

- Requires 2 I/Os: one for index page, one for heap page (to check visibility)
- \bullet Leaf-level is often non-contiguous on disk \rightarrow random I/O
- Unless index order matches heap order (clustered index), needs random I/O to read heap tuples as well

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Goal

Sort an arbitrary-sized relation using a fixed amount of main memory

- Arbitrary disk space
- Optimize to reduce I/O requirements
 - Not necessarily the number of comparisons!

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External Merge Sort

- Divide the input into runs, sort each run in-memory, write to disk
- Recursively merge runs together to produce longer sorted runs
- Eventually, a single run contains the entire sorted output

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External Sort, v1

```
-- run generation phase
while (t = getNext()) != NULL:
    add t to buffer
    if buffer exceeds work_mem:
        sort buffer
        write to run file, reset buffer
  merge phases
while > 1 run:
    for each pair of runs:
        merge them to into a single sorted run
```

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Optimization Goals

- \bullet Forms tree: height is # of merge phases, leaf level is # of initial runs
- $\bullet\,$ We read and write the entire input for each tree level $\rightarrow\,$ try to reduce tree height

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Replacement Selection

- Simple approach: read input until work_mem is reached, then sort and write to temp file
- Better: read input into an in-memory heap. Write tuples to temp file as needed to stay under work_mem
 - Next tuple to be written to a run is the smallest tuple in the heap that is greater than the last tuple written to that run
- Result: more comparisons, but runs are typically twice as large

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Optimizing The Merge Phases

Simple Approach

Merge sorted runs in pairs, yielding a binary tree (fan-in = 2)

• To reduce tree height, maximize fan-in: merge > 2 runs at a time

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Better Approach

- **1** Read the first tuple from each input run into an in-memory heap
- Repeatedly push the smallest tuple in the heap to the output run; replace with the next tuple from that input run

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Optimizing I/O

- Very sub-optimal I/O pattern: random reads from input runs
- Therefore, use additional work_mem to buffer each input run: alternate between prereading to fill inputs and merging to write output
- Tradeoff: larger buffers optimizes I/O, but reduces fan-in

Don't Materialize Final Merge Phase

Skip final merge phase: produce output from the penultimate set of runs

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Small Inputs

Many sorts are small \rightarrow just buffer in work_mem and quicksort

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Avoid Redundant Sorts

If the input is already sorted, we can avoid the sort altogether

• A sizable portion of the planner is devoted to this optimization ("interesting orders")

A Special Case: LIMIT

Can we do better, if we know at most k tuples of the sort's output will be needed?

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8.3 Feature

- If k is small relative to work_mem, no need to go to disk at all
- Instead, keep k highest values seen-so-far in an in-memory heap
- Benefits:
 - No need to hit disk, even for large inputs
 - $O(n \cdot \log k)$ comparisons rather than $O(n \cdot \log n)$

Hash Join

Classic Hash Join Algorithm

```
For a HJ between R and S on R.k = S.k:
```

```
-- build phase
for each tuple r in R:
    insert r into hash table T with key r.k
-- probe phase
for each tuple s in S:
    for each tuple r in bucket T[s.k]:
        if s.k = r.k:
            emit output tuple (T[s.k], s)
```

Pick R to be the smaller input.

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The Problem What if we can't fit all of R into memory? Neil Conway. (Truviso) Query Execution in PostgreSQL October 20, 2007 29 / 42

Simple Algorithm

```
for each tuple r in R:
    add r to T with key r.k
    if T exceeds work_mem:
        probe S for matches with T on S.k
        reset T
-- final merge phase
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```

The Problem

- Works fairly well, but reads S more times than necessary
- If we're going to read S multiple times, we can do better

Algorithm

- Choose two orthogonal hash functions, h_1 and h_2
- Read in *R* and *S*. Form *k* partitions by hashing the join key using *h*₁ and write out the partitions
- Then hash join each of the k partitions independently using h_2
 - Two matching tuples *must* be in the same partition
 - If a partition does not fit into memory, recursively partition it via h_3

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Problems

- Sensitive to the distribution of input data: partitions may not be equal-sized
 - Therefore, we want to maximize k, to increase the chance that all partitions fit in memory
- Inefficient if R fits into memory: no need to partition at all

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Hybrid Hash Join

A Small But Important Refinement

- Treat partition 0 specially: keep it in memory
- Therefore, divide available memory among partition 0, and the output buffers for the remaining *k* partitions

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Partition Sizing

- If we have B buffers in work_mem, we can make at most B partitions
- If any of the partitions is larger than B, we need to recurse
- Tradeoff: devote more memory to partition 0, or to maximizing the number of partitions?

Neat Trick

When joining on-disk partitions, if $|S_k| < |R_k|$, switch them

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Basic Task

- Form groups ("map")
 - Collect rows with the same grouping key together
- ② Evaluate aggregate functions for each group ("reduce")

Similar techniques needed for duplicate elimination (DISTINCT, UNION).

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Aggregate API

For each aggregate, in each group:

- s = initcond
- **2** For each value v_i in the group:

•
$$s = sfunc(s, v_i)$$

inal = ffunc(s)

Simple Idea

Take the inputs in order of the grouping key

- Sort if necessary
- Is For each group, compute aggregates over it and emit the result

Naturally pipelined, if we don't need an external sort.

Grouping by Hashing

Simple Idea

- Create a hash table with one bucket per group
- Por each input row:
 - Apply hash to find group
 - Update group's state value accordingly

Inherently non-pipelinable. Typically performs well for small numbers of distinct groups.

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The Problem

What happens if the size of the hash table grows large?

- That is, if there are many distinct groups
- At present, nothing intelligent the planner does its best to avoid hashed aggregation with many distinct groups
- FIXME

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Two Requirements

- Duplicate elimination (unless ALL is specified)
- Perform set operation itself: UNION, INTERSECT, EXCEPT

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- Duplicate elimination (unless ALL is specified)
- Perform set operation itself: UNION, INTERSECT, EXCEPT

Implementation

Both requirements can be achieved by concatenating the inputs together, then sorting to eliminate duplicates

- For UNION ALL, we can skip the sort
- TODO: consider hashing?
- TODO: consider rewriting set operations \rightarrow joins

A Nice Idea, due to Graefe, Linville and Shapiro (1994)

- Both algorithms are simple for small inputs
 - Quicksort, classic hash join

 Use divide-and-conquer for large inputs: partition, then merge Hashing: partition on a *logical key* (hash function), then merge on a *physical key* (one partition at a time) Sorting: partition on a *physical key* (position in input), then merge on a *logical key* (sort key)

- I/O pattern: hashing does random writes and sequential reads, whereas sorting does random reads and sequential writes
- Hashing can be viewed as radix sort on a virtual key (hash value)

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• EXPLAIN pretty-prints the plan chosen for a query

- For each plan node: startup cost, total cost, and result set size
- Estimated cost is measured in units of disk I/Os, with fudge factors for CPU expense and random vs. sequential I/O
- A node's cost is inclusive of the cost of its child nodes
- EXPLAIN ANALYZE also runs the query and gathers runtime stats
 - Runtime cost is measured in elapsed time
 - How many rows did an operator actually produce?
 - Where is the bulk of the query's runtime really spent?
 - Did the planner's estimates actually match reality?
- Most common planner problem: misestimating result set sizes
 - When debugging for planner mistakes, work from the leaves up
 - (And of course, be sure to run ANALYZE)

```
SELECT t2.id, t1.name
FROM t1, t2
WHERE t1.tag_id = t2.tag_id
AND t2.field1 IN (5, 10, 15, ...)
AND t2.is_deleted IS NULL;
```

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```
Merge Join (cost=18291.23..21426.96 rows=3231 width=14)
            (actual time=14.024..212.427 rows=225 loops=1)
 Merge Cond: (t1.tag_id = t2.tag_id)
  -> Index Scan using t1_pkey_idx on t1
         (cost=0.00..2855.74 rows=92728 width=14)
         (actual time=0.041..115.231 rows=54170 loops=1)
  -> Sort (cost=18291.23..18299.31 rows=3231 width=8)
            (actual time=13.967..14.289 rows=225 loops=1)
        Sort Key: t2.tag_id
        Sort Method: quicksort Memory: 26kB
        -> Bitmap Heap Scan on t2
                 (cost=5659.07..18102.90 rows=3231 width=8)
                 (actual time=12.731..13.493 rows=225 loops=1)
              Recheck Cond: ((field1 = ANY ('{5, 10, 15, ...}'::integer[]))
                        AND (is_deleted IS NULL))
              -> Bitmap Index Scan on t2_field1_idx
                      (cost=0.00..5658.26 rows=3231 width=0)
                      (actual time=12.686..12.686 rows=225 loops=1)
                    Index Cond: (field1 = ANY ('{5, 10, 15, ...}'::integer[]))
Total runtime: 212,939 ms
```

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Thank you.

Any questions?

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Classic survey paper on query evaluation techniques:

• G. Graefe. Query Evaluation Techniques for Large Databases. In ACM Computing Surveys, Vol. 25, No. 2, June 1993.

The duality of sorting and hashing, and related ideas:

• G. Graefe, A. Linville, L. D. Shapiro. Sort versus hash revisited. *IEEE Transactions* on *Knowledge and Data Engineering*, 6(6):934–944, December 1994.

Hybrid hash join:

• L. D. Shapiro. Join Processing in Database Systems with Large Main Memories. *ACM Transactions on Database Systems*, Vol. 11, No. 3, 1986.

Postgres' external sorting implementation is based on Knuth:

• D. Knuth. *The Art of Computing Programming: Sorting and Searching*, vol. 3. Addison-Wesley, 1973.

An exhaustive survey on DBMS sorting techniques:

• G. Graefe. Implementing Sorting in Database Systems. In ACM Computing Surveys, Vol. 38, No. 3, 2006.

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